A PROBABILISTIC ECONOMIC ORDER QUANTITY (EOQ) MODEL FOR INVENTORY MANAGEMENT OF DRUGS AND HOSPITAL CONSUMABLES

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Abstract: The quality of the healthcare service of the Central Pharmacy in Benue State University Teaching Hospital (BSUTH) is constantly threatened by high demands placed by the various sub-pharmacies due to high influx of patients needing diverse healthcare service. The provision of solution to the aforementioned inventory problems in this tertiary healthcare system is not only relevant but a timely intervention. This was achieved in this work via the application of the Probabilistic EOQ model in determining the Economic Order Quantity (EOQ) and Re-order level (ROL) for each drug and hospital consumable. The result of the sensitivity analysis provides the hospital management with a distribution of the response of the economic order quantity and reorder level of each drug and hospital consumable to changes in their respective ordering costs. For a particular drug (D1), the EOQs and Reorder levels are 11 & 864, 24 and 810 and 34 & 786 units for the ordering costs of 2.5, 12.5 and 25 percent of drug actual costs per unit. It is hoped that this distribution will assist the hospital management in taking guided decision on the quantity of drug or hospital consumable that should be ordered, given the associated ordering cost. The study recommended that the model be used in the inventory management and planning of the Central Pharmacy of the Benue State University Teaching Hospital (BSUTH) Makurdi, Benue State.

Keywords: Inventory, model, probabilistic

Introduction

The pharmacy is one of the most extensively used therapeutic facilities of the hospital and one of the few areas where a large amount of money is spent on purchases on a recurring basis (Kunder*et al.,* 2000). Research have shown that about one-third of the annual expenditure budget is spent on buying materials and supplies including drugs and that medicines and medical supplies are the major portions of the hospital expenses (Kant *et al.*, 1997, Manhas*et al.*, 2012). This emphasizes the need for planning, designing and organizing the pharmacy to facilitate efficient healthcare service delivery. In another light, drug shortages can unpleasantly affect drug treatment, delay medical procedures and may result in medication error (Fox *et al.*, 2009). Thus, inventory management system should be developed in a cost effective and efficient way having the appropriate amounts of materials in the right place, at the right time and at low cost to strike balance between 'too much and too little'.

The aforementioned problem of drug shortage and its negative consequence is a picture of the problems currently facing the Benue State University Teaching Hospital (BSUTH) Makurdi, Nigeria. The hospital was established in 2010 by the Benue State Government of Nigeria and it commenced clinical services on the 26th of March, 2012. One of the main objectives of the hospital is to respond promptly and effectively to the needs of patients. It is the referral hospital for the various Local Government Areas in the state. It also, receives patient from General Hospitals and neighboring states of the federation. It is first of its kind state university teaching hospital in the Middle-Belt of Nigeria.

The quality of the healthcare service of the Central Pharmacy in the Benue State University Teaching Hospital (BSUTH) is constantly threatened by the high demands placed by the various sub-pharmacies due to the high influx of patients needing diverse healthcare services. The sub-pharmacies are General Out-Patient department (GOPD), Specialty/staff clinic pharmacy, Accident and Emergency pharmacy (A and E pharmacy), In-patient pharmacy and Chest clinic pharmacy. One among the many ugly incidence experienced in the hospital, is a situation where a patient is been ask to go outside the hospital premises at odd times of the night to get prescribed drugs or needed hospital consumables for emergency situations. The reason for an ugly situation as this is that no scientific method currently exists in determining the Economic Order Quantity (EOQ) and Reorder level of each drug and hospital consumable so as to ensure no stock-out, when demands are placed by the sub-pharmacies in the hospital. It is important to mention that the consequence of patients being ask to go outside the hospital premises to buy drugs or consumables is the loss of revenue to the hospital and the more subjective cost of loss in goodwill and frustration on the part of patients.

Therefore the provision of solution to the aforementioned inventory problems in this tertiary healthcare system is not only relevant but a timely intervention. This was achieved in this work via determination of the Economic Order Quantity (EOQ) for each drug and hospital consumable, Reorder level for each drug and hospital consumable and a sensitivity analysis to see how the EOQ and Reorder levels respond to changes in model parameters. These results were presented for each drug and hospital consumable to enhance proper pharmacy planning and administration.

This paper seeks to apply the probabilistic EOQ inventory model. The model is preferred to the Deterministic and ProbabilitizedEOQ Model. The reason been that most real life inventory systems have stochastic demands with high coefficient of variation (over 20%) thus Deterministic Inventory Models will not apply, while the ProbabilitizedEOQ Model will not yield optimal result because the probabilistic nature of demand is initially ignored only to be revived in a totally independent manner at a later stage. The probabilistic EOQ Model like other Inventory Models, seeks to find answer to two basic questions; how much to order (the EOQ)? and when to order (the Re-order level)? Some related works include one that focused on the development of a probabilistic continuous review inventory model with constant units of cost where the lead-time demand is a random variable (Hadley and Whitin, 1963), another work which studied the probabilistic single item, single source (SISS) inventory system with zero lead time via the classical optimization approach (Fabrycky and Banks, 1967).

Further in this light, a probabilistic multi-item inventory model with varying order cost, zero lead-time demand under two restrictions and no shortage was introduced (Abou-El-Ata *et al.*, 2003). In addition, several continuous distributions for constrained probabilistic lost sales inventory models with varying order cost were applied in solving inventory problems via the Lagrangian method (Fergany and El-Wakeel, 2006a,b). Another work in this direction include modified a geometric programming approach in order to determine the inventory policy of a probabilistic single-item Economic Order Quantity (EOQ) model that has varying order cost and zero lead time (Kotb and Al-Shanbari, 2011). Still on the list, is a work that studied the probabilistic continuous review inventory model with two shortage costs (El-Wakeel and Fergany, 2013). Some probabilistic EOQ inventory models have been applied to deteriorating items. These include one with ramp type demand rate, partial backlogging and time varying holding cost (Karmakar and Chondhuri, 2014), another with trade credit policy and variable deteriorating rate for fixed life time products (Sarkar*et al.*, 2014). Others include a two warehouse partially backlogging inventory model for deteriorating items with ramp-type demand rate (Kumar *et al.*, 2015) and an inventory model with ramp type demand rate under inflation (Bushil and Rajput, 2016).

Materials and Methods

Source of data

Quantitative data about the pattern of demand of drugs and hospital consumables were sourced from the existing records of the Benue State University Teaching Hospital (BSUTH) Makurdi for the period of two years (2013 to 2014). The drugs and hospital consumables as well as their respective codes are shown in Table 1.

Probabilistic EOQ model

There is no reason to believe that the "Probabilitized" EOQ model will produce an optimal inventory policy. The fact that pertinent information regarding the probabilistic nature of demand is initially ignored, only to be "revived" in a totally independent manner at a later stage, is sufficient to refute optimality (Taha, 2007). Therefore; we adopt the Probabilistic EOQ Model for this research work. This section focuses on the model description and mathematical background.

Model description and equations

The model policy calls for ordering the quantity y whenever the inventory drops to level R. The optimal values of y and R are determined by minimizing the expected cost per unit time which is defined as the sum of the setup, holding, and shortage costs.

Assumptions and notations

The following assumptions and notations will be adopted in deriving the inventory model.

Assumptions

The model has three assumptions:

Unfilled demand during lead time is backlogged.

No more than one outstanding order is allowed.

The distribution of demand during lead time remains stationary (unchanged) with time.

Notations

To develop the total cost function per unit time, let

f(x) = probability density function of demand, x, during lead time

D = Expected demand per unit time

h = Holding cost per inventory unit per unit time

p = Shortage cost per inventory unit

K = Ordering cost or setup cost per order

Based on these definitions, the elements of the cost function are now determined.

Setup cost: the approximate number of orders per unit time is $\frac{D}{y}$, so that the setup cost per unit time is approximately;

$$\frac{KD}{y} \qquad (1)$$

Expected holding cost: the average inventory is;

$$\text{I} = \frac{(y+E\{R-x\})+E\{R-x\}}{2} = \frac{y}{2} + R - E\{x\} \qquad (2)$$

The formula is based on the average of the beginning and ending expected inventories of a cycle, $y + E\{R - x\}$and$E\{R - x\}$, respectively. As an approximation, the expression ignores the case where $R - E\{x\}$ may be negative. The expected holding cost per unit time thus equals

$$hI \qquad (3)$$

Expected shortage cost: Shortage occurs when $x > R$. Thus, the expected shortage quantity per cycle is;

$$S = \int_R^\infty (x-R)f(x)dx \qquad (4)$$

Because p is assumed to be proportional to the shortage quantity only, the expected shortage cost per cycle is pS, and, based on $\frac{D}{y}$ cycles per unit time, the shortage cost per unit time is;

$$\frac{pDS}{y} \qquad (5)$$

The resulting total cost function per unit time is;

$$TCU(y,R) = \frac{DK}{y} + h\left(\frac{y}{2} + R - E\{x\}\right) + \frac{pD}{y}\int_R^\infty (x-R)f(x)dx \qquad (6)$$

The solutions for optimal y^* and $R^*$are determined from by taking the partial derivative of equation (6) with respect to y and then with respect to R. This yield;

$$\frac{\partial \text{TCU}}{\partial y} = -\frac{DK}{y^2} + \frac{h}{2} - \frac{PD}{y^2}\int_R^\infty (x-R)f(x)dx \qquad (7)$$

Recall that $S = \int_R^\infty (x-R)f(x)dx$ and equating $\frac{\partial \text{TCU}}{\partial y}$ to zero yields;

$$\frac{\partial \text{TCU}}{\partial y} = -\left(\frac{\text{DK}}{\text{y}^2}\right) + \frac{\text{h}}{2} - \frac{\text{pDS}}{\text{y}^2} = 0 \qquad (8)$$

Solving for y yields the EOQ(y^*) as given in equation (9)

$$y^* = \sqrt{\frac{2D\ (K+pS)}{h}} \qquad (9)$$

Taking the partial derivative of equation (6) with respect to R and equating to zero, we have;

$$\frac{\partial \text{TCU}}{\partial \text{R}} = \text{h} - \left(\frac{\text{pD}}{\text{y}}\right)\int_R^\infty f(x)dx = 0 \qquad (10)$$

Solving yields,

$$\int_{R^*}^\infty f(x)dx = \frac{hy^*}{pD} \qquad (11)$$

Because y^* and R^* cannot be determined in closed forms from (9) and (11), a numeric algorithm, developed by Hadley and Whitin (1963), is used to find the solutions. The algorithm converges in a finite number of iterations, provided a feasible solution exists.

For $R = 0$, equation (9) and equation (11), above yield

$$\hat{y} = \sqrt{\frac{2D(K+pE\{x\})}{h}} \qquad (12)$$

and

$$\tilde{y} = \frac{PD}{h} \qquad (13)$$

If $\tilde{y} \geq \hat{y}$, unique optimal values of y and R exist. The solution procedure recognizes that the smallest value of $y^*$is

$\sqrt{\frac{2KD}{h}}$, which is achieved when $S = 0$.

The steps of the algorithm are

Step 0: Use the initial solution $y_i = y^* = \sqrt{\frac{2KD}{h}}$, and let $R_0 = 0$. Set $i = 1$, and go to step i.

Step i: Use y_i to determine R_i from Equation (11). If $R_i \approx R_{i-1}$, stop; The optimal solution is $y_i = y^*$, and $R^* = R_i$, otherwise, use R_i in equation (11) to compute y_i. Set $i = i + 1$, and repeat step i.

Estimate of model parameters

For holding cost, the cost components include cost of electricity from the Power Company, cost of generator (Diesel) usage and maintenance service. It also includes the air conditioner maintenance/replacement of sockets and bulbs. The ordering cost of drugs and hospital consumables is fixed at 20% of the actual item cost (an agreement between the hospital management and suppliers) while the shortage cost is simply the unit cost of each item. The distribution of the holding, ordering and shortage costs for each drug and hospital consumable were obtained as shown in Table 3. Estimate of holding cost for each drug was obtained using the quantity demanded per unit time of one (1) week and the total holding costs of items per week. Estimate of ordering and shortage cost where obtained as described above.

Sensitivity analysis

Sensitivity analysis were performed in order to determine the response of the EOQ (y^*) and Re-order level (ROL) to changes in model parameters most especially the ordering cost of drugs and hospital consumables. This is because; the pharmaceutical companies who supply products are agitating for increase in the ordering costs of their products. Thus, the hospital management is therefore faced with the challenge of determining ordering costs for each product that is optimal.

Use of software

The Predictive and Analytic Software (PASW) version 21 was employed in fitting demand distribution of drugs and hospital consumables while the Microsoft Excel package (2007) was employed in computing descriptive statistics and plotting of graphs.

Result and Discussion

This section is concerned with the discussions on the results of the descriptive statistics of the quantity of drugs and hospital consumables demanded per unit time and during the lead time of two weeks, the distribution fits for demand during lead time and the model parameter estimates for drugs and hospital consumables. It also includes the optimal economic order quantity (EOQ) and reorder level for each drug and hospital consumable and moreover, the result of the sensitivity analysis on ordering cost.

Estimates of model parameters

The estimate for the holding, ordering and shortage cost, were made from data and expert opinions. For holding cost, the costs of Electricity (from the Power Company), Generator (Diesel) use, it's maintenance service as well as cost of servicing air conditioner and replacements of sockets and bulbs where used. A close interaction with experts in the maintenance department of the hospital revealed that, the total electric power of 8KVA per month amounting to #14,857 from direct electricity and #8,000 per month from Generator use. Generator maintenance cost is estimated as #25,000 per month. Furthermore, the cost of Air conditioner maintenance and replacement of sockets and bulbs per month is estimated as #2,500 (Table 1). As earlier mentioned, the holding cost for each drug and hospital consumable is obtained using the quantity demanded per unit time of one (1) week and the total holding costs of items per week and displayed in Table 2. The ordering cost of each drug and hospital consumables is fixed by a binding agreement between the hospital management and the pharmaceutical companies at 20% of the actual item cost, while the shortage cost is simply the unit cost of each drug and hospital consumable (Table 2).

Table 1: Estimate of holding costs

S/N	Cost Component	Cost Per Month (#)	Cost Per Week (#)
1.	Electricity (JED) (8KVA)	14,857	3,714.25
2.	Generator Diesel (8KVA)	8,000	2,000
3.	Generator Service/Maintenance	25,000	6,250
4.	Air conditioner maintenance and replacement of sockets and bulbs	2,500	625

Table 2: Model parameter estimates for drugs and hospital consumables

Drugs/Hospital Consumables Codes	Parameter Estimates		
	Holding Cost (#)	Shortage Cost (#)	Ordering Cost (#)
D1	18.86	504	50
D2	24.95	1,850	300
D3	32.05	1330	200
D4	21.37	1260	176
D5	3.51	100	15
D6	188.53	1,730	266
D7	16.36	520	46
D8	70.45	7500	594
D9	17.14	3000	22
C10	40.07	800	120
C11	16.40	1,100	160

Table 3: Descriptive statistics of quantity of drugs and hospital consumables demanded per week

Drugs/hospital consumables codes	Descriptive statistics of quantity demanded		
	Mean	Standard deviation	Coefficient of variation (%)
D1	170	195	57
D2	129	165	64
D3	100	122	61
D4	150	122	41
D5	913	1067	58
D6	17	27	79
D7	196	189	48
D8	46	61	68
D9	187	547	146
C10	80	221	138
C11	196	404	103

Descriptive statistics of quantity of drugs and hospital consumables demanded per unit time

The high standard deviation of the quantity of drugs and hospital consumables demanded per unit time indicates that demand is highly variable. The high coefficient of variation of over 20% also substantiates the stochasticity of this demand per unit time for each item. This is the reason why the probabilistic Economic Order Quantity (EOQ) model is preferred for use over the deterministic EOQ model (Table 3).

Descriptive statistics of quantity of drugs and hospital consumables demanded during lead time of two weeks

The policy of the Benue State University Teaching Hospital (BSUTH) is that demand placed on drug and hospital consumables should have a lead time of two (2) weeks. The lead time for each item was traced throughout the data span and found to be approximately two (2) weeks (Table 4). The descriptive statistics for demand during lead time for each item is specified by the mean and standard deviation for each item. The mean demand during lead time for each item is seen to vary across the items, while the standard deviation of item demand during lead time is seen to be high justifying as earlier mentioned, the selection of the probabilistic EOQ model for use in this research.

Table 4: Descriptive statistics of quantity of drugs and hospital consumables demanded during lead time (two week)

Drugs/Hospital Consumables Codes	Descriptive Statistics of Quantity Demanded		Average Lead Time (Days)
	Mean	Standard Deviation	
D1	300	200	13.7
D2	328	137	13
D3	150	58	13.5
D4	300	141	13
D5	2067	1144	13.3
D6	38	28	12.8
D7	331	128	13.2
D8	108	57	13
D9	168	158	11.4
C10	159	221	12.2
C11	860	859	12.9

Table 5: Distribution fit for demands during lead time

Drugs/hospital consumables codes	Distribution/Parameters	Value of Kolmogorov-Smirnov Statistic	P-Value
D1	Normal(300, 199.833)	0.485	0.973
D2	Normal(328,137.145)	0.501	0.963
D3	Normal(150, 57.735)	0.614	0.846
D4	Normal(300, 141.421)	0.520	0.949
D5	Normal(2067,1144.114)	0.488	0.971
D6	Normal(38, 28.293)	0.473	0.979
D7	Normal(331, 128.325)	0.456	0.985
D8	Normal(108, 56.789)	0.546	0.927
D9	Normal(168, 157.861)	0.505	0.961
C10	Normal(159, 221.140)	0.867	0.439
C11	Normal(859, 858.795)	1.060	0.211

Fitted distribution of the demand during lead time

The demand data of the quantity of drugs and hospital consumables was used for the distribution fit. The normal distribution was ascertained to fit the nature of demands. Table 5 shows the distribution fits and the parameter details. Normal distribution fit to demand data, was also used by Hadley and Whitin (1963), Fabrycky and Banks (1967) and El-wakeel and Fergany (2013) in their probabilistic EOQ model.

Economic order quantity and reorder level of drugs and hospital consumables

The computed value of Economic Order Quantity (EOQ) and reorder level of each drug and hospital consumable is shown in Table 6. The table revealed that the optimal inventory policy is to order 31 sachets of lisinopril tablets (D1) whenever the inventory level falls to 794 sachets, 56 vial of rocephine injection (D2) whenever the inventory level falls to 673 vial, 35 sachets of amoxicillin + clavulanic acid tablet (D3) whenever the inventory level falls to 288 sachets, 50 sachets of clopidogrel tablet (D4) whenever the inventory level falls to 657 sachets, 88 bottles of metronidazole infusion (D5) whenever the inventory level falls to 5,167 bottles, 7 bottles of lactulose suspension (D6) whenever the inventory level falls to 86 bottles, 33 bottles of aluminum hydroxide + magnesium semithicone suspension (D7) whenever the inventory level falls to 657 bottles, 28 sachets of amiloride + hydrochlorothiazide tablet (D8) whenever the inventory level falls to 252 sachets, 22 bottles of ringer lactate (D9) whenever the inventory level falls to 252 bottles, 22 rolls of cotton wool (C10) whenever the inventory level falls 645 rolls and 62 packs of examination gloves (C11) whenever the inventory level falls to 3,093 packs.

Table 6: A distribution of the response of EOQ and ROL of drugs and hospital consumables to changes in ordering costs

Drugs/hospital consumables codes	2.5%		5%		7.5%		10%		12.5%		15%		17. 5%		20.5%		22.5%		25%	
	EOQ	ROL	EOQ	ROL	EOQ	ROL	EOQ	ROL	EOQ	ROL	EOQ	ROL	EOQ	ROL	EOQ	ROL	EOQ	ROL	EOQ	ROL
D1	11	864	15	842	19	828	22	820	24	810	27	804	29	798	31	794	32	790	34	786
D2	20	720	28	706	34	697	39	690	44	684	48	680	52	677	56	673	59	671	62	668
D3	12	310	18	303	22	298	25	296	28	293	31	291	33	290	35	288	37	287	39	286
D4	18	706	25	691	30	682	35	674	39	669	43	664	46	660	50	657	53	654	56	651
D5	31	5533	44	5149	54	5339	62	5282	70	5247	77	5224	83	5190	88	5167	94	5144	99	5121
D6	2	98	3	94	4	92	5	90	5	89	6	88	6	86	7	86	7	85	8	84
D7	12	702	17	687	20	679	23	673	26	668	29	664	31	660	33	657	35	655	37	652
D8	10	272	14	266	17	262	20	259	22	257	24	255	26	253	28	252	30	251	31	250
D9	8	719	11	704	13	694	15	685	17	685	19	677	20	677	22	670	23	670	24	670
C10	8	731	11	703	13	687	15	674	17	665	19	658	20	652	22	645	23	641	24	636
C11	22	3377	31	3282	38	3231	44	3196	49	3162	54	3128	58	3111	62	3093	66	3068	69	3050

Economic Order Quantity (EOQ); Re-order level (ROL)

The sensitivity analysis

From the results of the sensitivity analysis on the response of the economic order quantity and reorder level to changes in the ordering cost as shown in Table 6, it can be inferred that while the economic order quantity of each drug and hospital consumable increases with increase in their ordering cost, the reorder level decreases with increase in the ordering cost (Figs. 1 – 8). As earlier stated in this work; that the pharmaceutical companies are agitating for increase in the ordering costs of drugs and hospital consumable, it is therefore necessary for the hospital management to have a distribution of the response of the economic order quantity and reorder level of each drug and hospital consumable to changes in their respective ordering costs. This distribution is given in Table 8 across varying values of ordering cost. It is hoped that this distribution will assist the hospital management in taking guided decision on what quantity of drug or hospital consumable that should be ordered given the associated ordering cost.

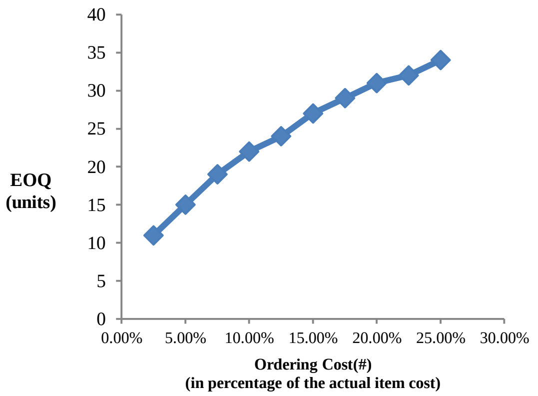

Fig. 1: Optimal values of EOQ against ordering cost for Lisinopril tablet (10 mg)

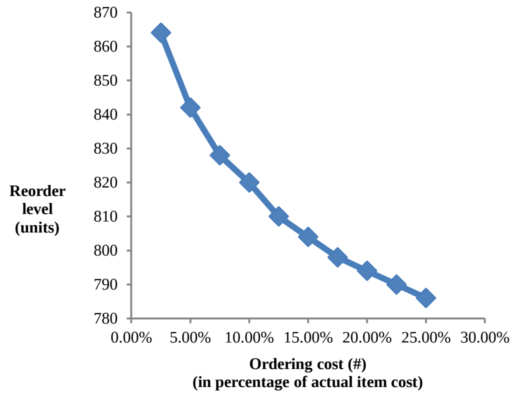

Fig. 2: Optimal values of reorder level against ordering cost for Lisinopril tablet (10 mg)

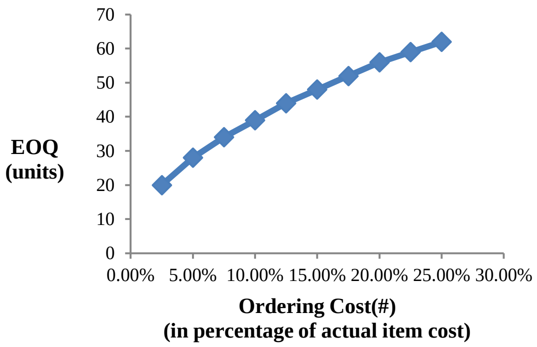

Fig. 3: Optimal values of EOQ against ordering cost for Rocephine injection (Ceftriaxone) 1 g

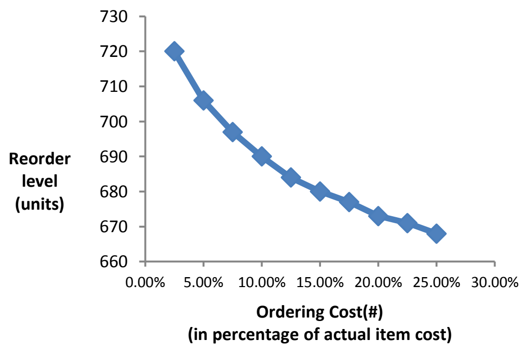

Fig. 4: Optimal values of reorder level against ordering cost for Rocephine Injection (Ceftriaxone) 1 g

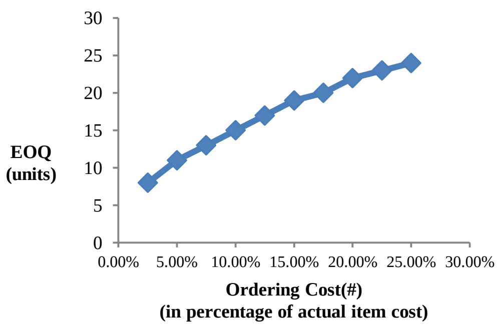

Fig. 5: Optimal values of EOQ against ordering cost for cotton wool (500 g)

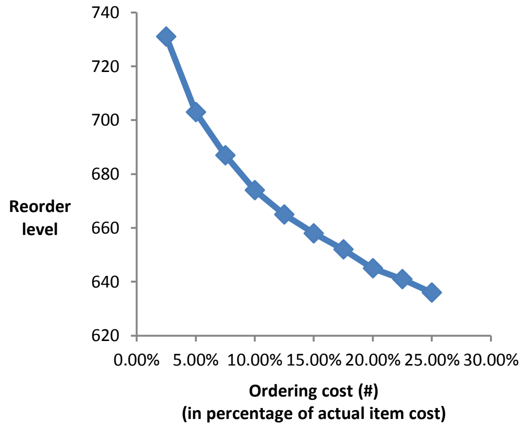

Fig. 6: Optimal values of reorder level against ordering cost for cotton wool (500 g)

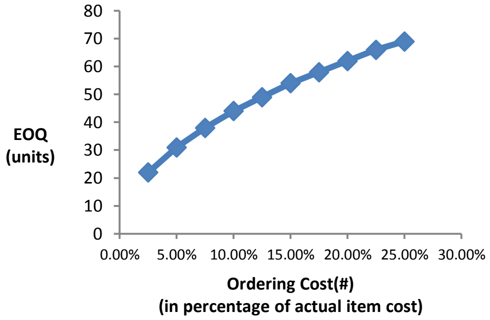

Fig. 7: Optimal values of EOQ against ordering cost for examination gloves

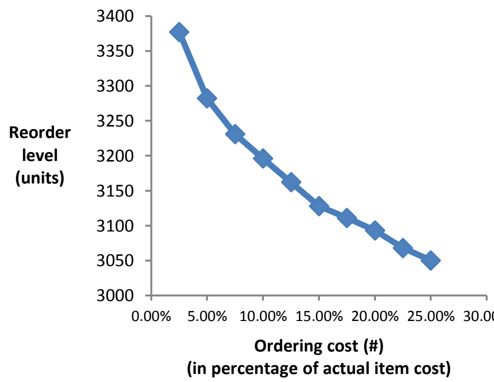

Figure 8: Optimal values of reorder level against ordering cost for examination gloves

Conclusion

From the results of the study, the following conclusions were drawn: That the probabilistic EOQ model has been able to determine the optimal economic order quantity and reorder level of each drug and hospital consumable required in the Central Pharmacy of the Benue State University Teaching Hospital (BSUTH). That the Economic Order Quantity (EOQ) of each drug and hospital consumable increases with increase in the ordering cost while the Re-order decreases with increase in the ordering cost. That a distribution of the response of the EOQ and Re-order level of each drug and hospital consumable to changes in their respective ordering costs will help the hospital management in making guided decisions.

Recommendation

This model should be used in the inventory management and planning of the Central Pharmacy of the Benue State University Teaching Hospital (BSUTH) Makurdi, Benue State.For comparative analysis, a simulation model of the Inventory System of the Benue State University Teaching Hospital (BSUTH) Central Pharmacy is recommended for future research.

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